EVALUATING THE TIPPING POINT OF A COMPLEX SYSTEM: THE CASE OF DISRUPTIVE TECHNOLOGY

Abstract

Complex systems often operate in equilibriums that can be disrupted under specific conditions, and disruption may drive the system to undergo an irreversible phase transition (aka tipping point) into a new equilibrium. Disruptive technology, as introduced in the book Innovator's Dilemma, is a new emerging technology that can successfully displace the incumbent technologies and, as a result, push the market (as a complex system) through phase transition into a new equilibrium. In this paper, the authors model the market disruption caused by a disruptive technology as a complex system, with dynamics that show a phase transition, or tipping point, after which the system shifts into a new equilibrium aiming at assessing the success or failure of a disruptive technology. The purpose of the current study is to mathematically model a tipping point measure of complex networks and a theoretical framework for disruptive technology dynamics. A predator-prey model is used to emulate the behavior of a disruptive technology versus an incumbent technology in the market dynamics. A resilience index is integrated into the model to measure the tipping point in the market where the disruptive technology will overtake the incumbent technology. This methodology is then applied to a historical case study of film vs. digital vs. cell phone cameras, which is simulated to demonstrate the application of this methodology. The contribution of the current study can be applied to both systems engineering and disruptive innovation management. Additionally, the proposed novel approach can help stakeholders of similar complex economic systems to assess the impact of a potential new disruptive technology, and potentially use the resulting resilience index as a measurement for adjusting technology requirements and systems management approaches to achieve a desired outcome.

Keywords: Complex systems, Complexity Theory, Tipping Point, Phase transition, Predator-prey model, disruptive technology

1. INTRODUCTION

In recent decades, the field of systems engineering has been grappling with the growth in the complexity of new cyber-physical systems. Complex systems are often composed of multiple components interacting in non-trivial and nonlinear ways and through a continuous interplay between the systems and their environment [1], [2]. Growth in the complexity of a system is often inevitable. An example of this might be the IT Systems, which have seen a significant growth in complexity. This has been mostly due to an exponential rise in consumers' adoption and use of new technologies, coupled with organizations trying to keep pace with technological advancement by deploying even more technologies. However, a systems engineer's goal is to design a system such that its behavior is predictable in the face of external and environmental changes [3]. The external changes consist of factors that originate from outside the system and are beyond an organization's grasp and control, while environmental changes are more concerned with the depletion of natural resources and environmental wastes, among others. The growth in complexity of a system inevitably leads to its unpredictable behavior [4, 5]. Therefore, modeling complexity becomes a challenge for natural and engineered systems. A major reason behind the unpredictable behavior of a complex system is its phase transitions, or tipping points, which are a challenge to manage in the systems engineering domain. A tipping point or phase transition can be stated as a particular point where one or more external stressor(s) or change in variables lead to interrupting the steady state of the system performance [6]. In the context of a complex system, a tipping point is a qualitative change in a system operation mathematically represented by bifurcation [7]. For example, the qualitative change in an economic system can be a new competitive entrant that disrupts a market's status quo. Then the mathematical representation of those changes can be a bifurcation between stability and instability of the incumbent companies' market shares. The precise moment where a complex system can undergo an abrupt and unanticipated shift from one state to another is often difficult to evaluate and predict [8, 9]. Furthermore, this shift is often irreversible in nature and leads to the creation of a new system [10]. Therefore, this research paper aims to introduce a preliminary approach to modeling the tipping point of a complex system involving disruptive technology in its market. The authors propose a modified resiliency metric that enables decision-makers to assess the chance of a novel technology in the market becoming disruptive. Figure 1 illustrates how the resilience metric approach is used when it is applied to a representative system. It illustrates the system's stability using a mathematical dimension named the effective plane and a resilience index, β_{eff}, to measure a complex system's precariousness, or how close it is to its tipping point at β_{eff}^{c}[11]. Disruptive technology often gradually gains a foothold in the mainstream market and ultimately replaces it over time. Because disruptive technology causes a tipping point in market behavior, these market dynamics can be modeled and analyzed using this method.

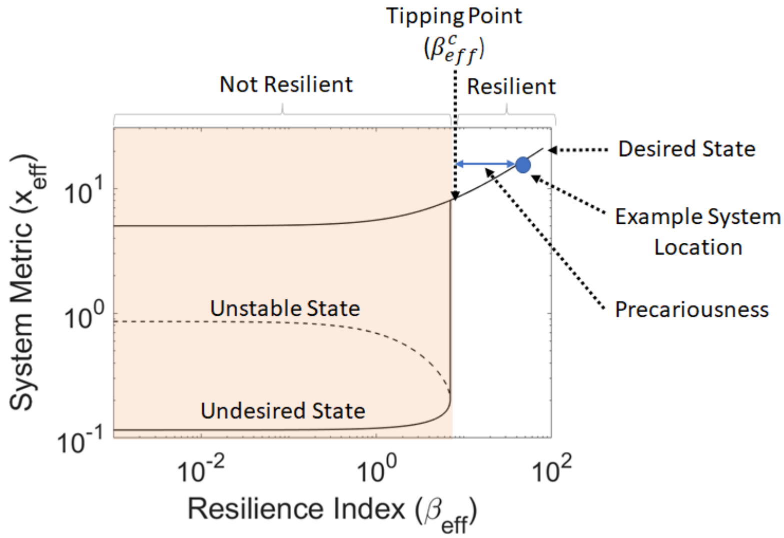

Figure 1. Example of effective plane and resilience index [12]

This paper is a research effort to mathematically model the tipping point of a complex system, with the specific aim of assessing a potential disruptive technology as a case study. The study also aims to assess disruptive technology's potential success or failure in the mainstream market. The paper begins with reviewing a summary of the state-of-the-art literature on complex systems, tipping point, and disruptive technology. Next, the authors propose a model to evaluate the tipping point of a disruptive technology, which, in the context of the current study, is considered a complex system. The tipping-point measure in the paper is based on the Lotka-Volterra model of biological ecosystems [13] translated into the mathematical dimension called the effective plane that measures inflection points using a resilience index [11]. Building on the Lotka-Volterra model of technology disruption [14], this translated model uses a case study of comparing film cameras vs. digital cameras in cell phones an illustrative case. Next, the simulation results are discussed and compared to the historical data of this case study. The paper concludes with a discussion of the results, conclusions, and future work. The contribution of the current study can be applied to both systems engineering as well as disruptive innovation management.

2. LITERATURE REVIEW

In this section, a brief overview of selected literature related to tipping point/phase transition and disruptive technology is summarized.

2.1 Complexity, Tipping Point, and Phase Transitions

Complexity is one of the main characteristics of many large-scale natural and engineered systems. Complex engineered systems can provide sophisticated functionalities. However, they are prone to emergent behaviors and increased fragility [15, 16]. The current study adopts the definition of complexity by Willcox [17] as "… the potential of the system to exhibit unexpected behavior. Complex systems behaviors are often difficult to predict, and interactions between subsystems and components are often non-linear. Non-linear behavior and emergence are among the main characteristics of complex systems. Over the last few decades, diverse research has been initiated to define, categorize, characterize, and measure complexity in various domains spanning from biology to physics and engineering [2, 17-21].

Complex systems can often experience tipping points or go through phase transitions. The term phase transitions, and tipping points are sometimes used interchangeably. They are thresholds or states where the entire complex system loses its stability permanently, without the possibility of reverting to any prior, or even the original state. Popularized by Gladwell in his book "The Tipping Point" [22], the concept of tipping point assumed a position in academic literature much earlier, in various fields spanning from mathematics and physics to psychology and social sciences. One of the earliest studies on tipping point was conducted by Clotfelter [23] in the domain of social sciences. Granovetter [24], in his study concerning threshold models, used the term 'threshold' as the point when a transition occurs (tipping point). This was further supported by his study on threshold and collective behavior conducted in 1983 [25]. Other notable early works on tipping point included the study by Schelling [26] on segregation and Crane [27] on neighborhood effects on dropping out and teenage childbearing.

A tipping point is a critical threshold where the introduction of small disturbances causes changes in the state of the system that may be entirely disproportional to the cause. At times, such disturbances can lead to an irreversible system collapse [28, 29]. The tipping point is not necessarily the exact point at which the major change is occurring but rather the point at which the variables are altered significantly, often irreversibly, thereby causing the transition and leading to the complex system going through its threshold [30].

Van Nes et al. define two different types of tipping points [31]. The tipping point of a system can occur when (*i)* there is a change in the external conditions and (ii) the state of the system undergoes a change (Van Ness et al., 2016). The first type is associated with the concept of bifurcations described by Scheffer et al. [32], and are caused by critical outside influences on the complex system in which the transitions shift the state of the system into an entirely different state. The second type of tipping point is derived from the domain of evolution and ecology and is related to unstable equilibria [33, 34]. These equilibria represent spots in the landscape of possibilities at which a slope exists on each side, making them mathematic extrema. The authors describe the two different types as "tipping due to change in conditions" and "tipping due to change in state." Van Nes et al. deduced a more general definition of the phenomenon of tipping points in a scientific way. They proposed to define tipping points as "any situation where accelerating change caused by positive feedback drives the system to a new state" [31]. Figure 2 provides the readers with tipping points and their accompanying changes.

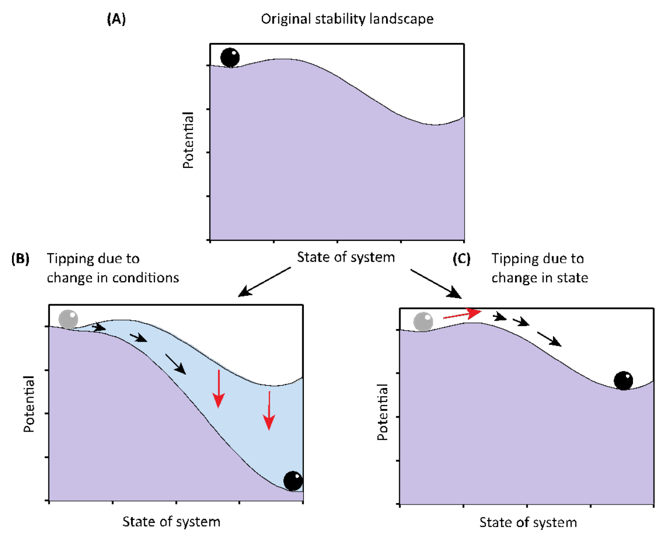

Figure 2. Tipping Points and accompanying changes [31].

The field of Systems Dynamics (SD) has also studied the phase transition/tipping point phenomena, and a tipping point can be stated as a threshold condition that might shift the dominance of the feedback loops that control a process [35]. Tipping points are conditions that border between two or more behavioral zones created by a dominant feedback loop [36]. The existence of a tipping point is evident that even when the system starts in the desirable execution mode, there is no guarantee that it will persist and stay stable [37]. Systems Dynamics can significantly aid in elucidating tipping points and their impacts on systems by specifying, formalizing, and explaining structures that create tipping points [38, 39]. The overshoot and collapse model was introduced in the seventies for systems dynamics [40] and has subsequently found a place in the literature of disruptive innovation as well [41, 42]. The overshoot and collapse model describes the behavior of a complex system for which a variable increase while consuming or eroding the carrying capacity of the environment, which can lead to the collapse of the system under study due to erosion or consumption of carrying capacity.

2.2 Disruptive Technology & Technology S-Curves: An overview

Disruptive innovations can be defined as "technologies that enable a new set of product features different from those associated with mainstream technologies and are initially inferior to the latter in certain attributes ('mainstream features') most valued by mainstream customers" [43]. Disruptive innovation is not just a mere technological attribute for an organization but is a concept that involves not only the new, emerging technology but the entire business model (and the system) in general, including customer requirements [44]. Disruptive technology can initially enter the market by concentrating on a low-end/niche segment, which the mainstream market incumbents find unattractive [45]. However, with time, the performance of disruptive technologies emulates and surpasses that of their incumbent counterpart and eventually 'invades' the mainstream markets [43, 45-48]. Disruptive innovations can create altogether new industries that eventually replace the existing ones [49, 50] – thereby following Schumpeter's [51] theory of creative destruction.

Every new technology matures in a unique timeframe and interacts with existing technologies in the market in various ways. Christensen [52] pioneered the definition of the transition that follows the technological S-Curves. These curves indicate the technical maturity and development of a technology or component over time, as well as the succession of various technologies in relation to each other.

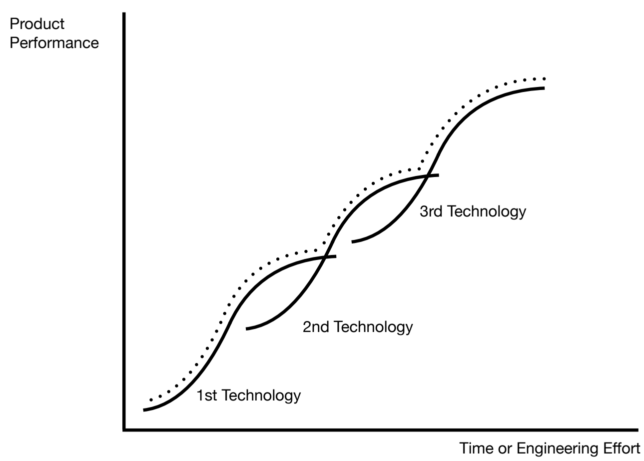

Figure 3. Prescriptive S-Curves [52]

As exhibited in figure 3, the S-curves follow each other in close succession and partially overlap. Figure 3 shows how three technologies follow in succession, with the performance of the product and technological maturity increasing over time. For each of those technologies, the advances are initially slow, especially right after its infancy, and slowly ramp up before reaching their peak growth. After this growth period, technology matures at a slower rate, and the advances slow down again. This opens the door for an alternative new technology that can now enter its phase of fast growth, with the objective of exceeding the performance and technological capabilities of the incumbent one. Eventually, a few new technologies could perform better than the initial technologies and compete for market share. This cycle then repeats itself with a third and successive technology, and so on. Various examples can be found for these phenomena such as the transition from incandescent lightbulbs to fluorescents, and then LED.

Products or services based on disruptive technology often follow a different value network than their incumbent counterparts. Value networks can be defined as “the contexts within which firms identify and respond to customer needs, solve problems, gather input, react to competitors, and seek profit” [53]. The initial level of performance for a disruptive product often lies well below that of the value network demanded by the existing customer base, and therefore, is neglected by the mainstream customer market. However, this new product/technology attracts the attention of a different set of customers - a set that was previously either ‘under-served’ or ‘over-served’ in the market [45, 54-57]. With the gradual advancement of time, it improves its performance to a level where the mainstream market segment finds its requirement being met by disruptive technology. Successful disruptive technologies, with time, reach a point where they surpass the existing incumbent technology in terms of their deliverables and functionalities. In the process of doing so, a successful disruptive technology not only initially addresses the needs of a niche segment of consumers but gradually evolves into addressing the requirements of the mainstream customer segment as well. This prompts the mainstream customers to switch to the new technology. As a result, the market incumbents often find themselves bereft of customers in the long- run and no longer relevant in that market.

In the context of disruptive innovation, incumbent firms have been observed to leave the market in search of more profitable customers, which in turn might lead to a collapse in the market, thereby creating a vacuum that attracts new entrants in the market [58]. On the other hand, oscillation pertains to the behavior of a second (or higher) order system where variable values increase and decrease in cycles/oscillations over time. If the amplitude is constant, there is no change in behavior, if it increases, the variables absolute grow over time, and vice versa [35, 59, 60]. The discrete nature of disruptive innovation might stand against the oscillating nature of the market, and thus, the timing of the disruptive innovation has to be considered [61].

3. RESEARCH APPROACH AND METHODOLOGY

The current study utilizes and modifies the network-based dynamics model of a system that has been proposed by Gao et al.[11] and combines it with an adapted version of the Lotka-Volterra equations of the predator-prey model. A previous model showed how the dynamics of disruptive technology in interaction with the mainstream market resembles predator-prey behavior. [14] The authors believe that this model can be modified using the resiliency and equilibrium calculation presented by Gao et al.[11] to measure market resiliency and potential tipping points due to disruptive technology. A brief overview of network-based model resiliency is summarized in section 3.1. Section 3.2 is devoted to the authors presenting the modified Lotka-Volterra equations of a predator-prey model to find the location of tipping points for a disruptive technology. Section IV presents a historical case study to demonstrate the application of the authors' theoretical development in part B of this section.

3.1 Background and Exiting Approach to Model Tipping Points

One approach for measuring location and distance from tipping points in a complex system starts with a network-based dynamics model of a system. Each node, *i*, has a differential equation for $\frac{dx_i}{dt}$ that describes the behavior of the system. In this equation, $F(x_i)$ models the self-dynamics of the node, and $G(x_i, x_j)$ models the effect of node j on node i, which is summed over N nodes. The adjacency matrix, $\boldsymbol{A}$, is weighted so that each element, A_{ij}, captures the impact that node *j* has on the dynamics of node *i*.

$$\frac{dx_i}{dt} = F(x_i) + \sum_{j=1}^{N} A_{ij} G(x_i, x_j) \tag{1}$$

Gao et al. introduced a method for reducing Equation 1 so that its inflection points can be calculated using a mathematical dimension called the effective plane. [11] This set of dynamics equations is translated into the effective plane using the following conversion.

$$\beta_{eff} = \frac{\mathbf{1}^T \boldsymbol{A} \boldsymbol{s}^{in}}{\mathbf{1}^T \boldsymbol{A} \mathbf{1}} \tag{2}$$

$$x_{eff} = \frac{\mathbf{1}^T \boldsymbol{A} \boldsymbol{x}}{\mathbf{1}^T \boldsymbol{A} \mathbf{1}} \tag{3}$$

For Equations 2 and 3, $\mathbf{1}$ is the unit vector $\mathbf{1} = (1, \dots, 1)^T$ and $\boldsymbol{s}^{in} = (s_1^{in}, \dots, s_N^{in})^T$ is the vector of incoming weighted degrees in adjacency matrix $\boldsymbol{A}$, with $s_i^{in} = \sum_{j=1}^{N} A_{ij}$.

Conceptually, β_{eff} is a scalar that measures how connected the network is by averaging the impact the network nodes have on each other. The impact that node j has on node i is A_{ij}, the total impacts on node i from the rest of the network is summed with the calculation of the incoming weighted degree s_i^{in}, and β_{eff} essentially calculates a network-wide average of $\boldsymbol{s}^{in}$. The dynamic variable x_{eff} portrays a combined description of the dynamics of the system in the effective plane. Through this conversion, the set of N number of differential equations is reduced to one differential equation.

$$\frac{dx_{eff}}{dt} = F(x_{eff}) + \beta_{eff} G(x_{eff}, x_{eff}) \tag{4}$$

The tipping points of the system are found by solving for the stability points of Equation 4. The critical resilience index located at those stability points is β_{eff}^{c}. A system's distance to its tipping points is measured by calculating its current β_{eff} and how far it is from β_{eff}^{c}. With this approach, $\beta_{eff} - \beta_{eff}^{c}$ becomes a "resilience index" that measures a system's distance from its tipping points. This measurement has been given the name "resilience index" in literature because it was initially invented to measure ecological resilience, which is the distance of an ecological system from its tipping points.[53] Some engineering literature refers to this distance of a system from its tipping points as a measurement of precariousness. [53]

3.2 Proposed Model to Measure the Tipping point of a Disruptive Technology

To analyze tipping points that are caused by technology disruption, the authors adopt the Lotka-Volterra equations, which are non-linear differential equations that model the dynamics of predator-prey relationships originally proposed by the biophysicist Alfred Lotka and mathematician Vito Volterra in 1925 [62]. The authors believe that a modified version of the predator-prey model can simulate the behavior of the competitive market that includes the incumbent as well as disruptive technology. Pielou [63] suggested a version of that model modified as follows:

$$\frac{dx_i}{dt} = a_i x_i + b_i x_i^2 - \sum_{j=1}^{n} A_{ij}\, x_i x_j \tag{5}$$

In equation (5), x_i represents the number of creatures in species *i*. Parameters a_i and b_i adjust the model to match the self-dynamics of growth/decline of *i*. The weighted adjacency matrix A_{ij} captures the relationship between species i and j. Ünver [14] performed a study of technology dynamics using this Lotka-Volterra model, in which he validated the model for use in this application and simulated the disruption of digital cameras in the film-camera industry.

For the assessment, the authors built on their previously published work by first comparing the relationship between an incumbent technology and a potential disruptive technology [48]. The approach we took for the previous publication limited the system to modeling one disruptive technology and one incumbent technology. Since then, we have modified the approach so that it can scale, model, and encompass multiple technologies. The new approach takes the following form:

$$\frac{dx_i}{dt} = a_i x_i + b_i x_i^2 + \sum_{j=1}^{n} A_{ij}\, x_i x_j \tag{6}$$

Where x_i represents the units produced using technology i. The adjacency matrix A_{ij} captures the sales that each technology is taking away from the other. The change in Equation 6 with respect to Equation 5 is that the weighted adjacency matrix, A_{ij}, can contain both positive and negative values. The authors observed in their previous study that positive values in the adjacency matrix model the predatory-like behavior of a disruptive technology, and negative values model the prey-like behavior of an incumbent technology.

Using a mathematical dimension called the effective plane introduced by Gao et al. [11] that weights each parameter by its impact on the system, this set of equations for each product i, up to n sets of equations, can be reduced to only one set of equations for the entire industry. The conversion uses the following:

$$\beta_{eff} = \frac{1^T A s^{in}}{1^T A 1} \tag{7}$$

$$x_{eff} = \frac{1^T A x}{1^T A 1} \tag{8}$$

Where $s^{in} = (s_1^{in}, \dots, s_u^{in})^T$ is the vector of incoming weighted degrees in matrix A. One difference between this approach and that of Gao et al. is that β_{eff} can be negative because we have allowed matrix A to contain negative values. The translation into the effective plane results in the following equations, with scaling parameters a and b.

$$\frac{dx_{eff}}{dt} = ax_{eff} + bx_{eff}^2 + \beta_{eff} x_{eff}^2 \tag{9}$$

The study introduces that this overall equation in the effective plane can be further decomposed to enable the study of the interactions of different parameters or subsystems. In this case, we want to assess the interaction of the incumbent (x_{prey}) and disruptive (x_{pred}) technologies. Note that x_{prey} and x_{pred} are vectors that can be populated to characterize the technologies with more predatory behavior and those with more prey behavior. Multiple prey technologies can be characterized with a vector $x_{prey} = [x_{prey1};\ x_{prey2}; \dots]$, and multiple predatory technologies can be characterized with a vector $x_{pred} = [x_{pred1};\ x_{pred2}; \dots]$ with an example provided in the second case study below. The system is subsequently decomposed using the following conversion equations.

$$\beta_{eff} = \frac{1^T A s^{in}}{1^T A 1} \tag{10}$$

$$x_{prey,eff} = \frac{1^T A \begin{bmatrix} x_{prey} \\ 0 \end{bmatrix}}{1^T A 1} \tag{11}$$

$$x_{pred,eff} = \frac{1^T A \begin{bmatrix} 0 \\ x_{pred} \end{bmatrix}}{1^T A 1} \tag{12}$$

Resulting in the following equations in the effective plane:

$$\frac{dx_{prey,eff}}{dt} = ax_{prey,eff} + bx^2_{prey,eff} + \beta_{eff} x_{prey,eff} x_{pred,eff} \tag{13}$$

$$\frac{dx_{pred,eff}}{dt} = ax_{pred,eff} + \mathrm{b}x^2_{pred,eff} + \beta_{eff} x_{pred,eff} x_{prey,eff} \tag{14}$$

The study found the stability points of these equations by setting $\frac{dx_{prey,eff}}{dt} = 0$ and $\left(\frac{dx_{prey,eff}}{dt}\right)_{dx_{prey,eff}} = 0$, or $\frac{dx_{pred,eff}}{dt} = 0$ and $\left(\frac{dx_{pred,eff}}{dt}\right)_{dx_{pred,eff}} = 0$, and solving for the critical β^c_{eff}. Following the method by Gao et al. [11], we hypothesize that when a system crosses one of these β^c_{eff}, it will undergo a phase transition, or a tipping point where its dynamical behavior will change.

$$\beta^{c0}_{eff} = 0 \tag{15}$$

$$\beta^{c1}_{eff} = -2b \tag{16}$$

$$\beta^{c2}_{eff} = b \tag{17}$$

$$\beta^{c3}_{eff} = 2b \tag{18}$$

The interpretation of a market going through phase transition translates into a higher chance for disruptive technology to take over and become established in the mainstream market. Similarly, if a new technology does not move the market equilibrium passed the tipping point, it reduces the chance of a new technology in the market becoming disruptive and taking hold.

The following section (section 4) uses a historical case study of film versus digital cameras to simulate the disruptiveness of digital cameras in the mainstream market at the time.

4. CASE STUDIES: FILM CAMERAS VS. DIGITAL CAMERAS VS. PHONES

To test the proposed new approach to assess the potential for a technology to be disruptive, we first applied it to a case study of the market interactions of film and digital cameras [11]. In this case, the digital cameras were the disruptive technology that overtook the sales of the incumbent technology, the film cameras. Using Equation 6, the dynamics equations that describe the predator-prey interactions of the digital and film sales have the following form:

$$\frac{dx_1}{dt} = a_1 x_1 + b_1 x_1^2 + A_{12} x_1 x_2 \tag{19}$$

$$\frac{dx_2}{dt} = a_2 x_2 + b_2 x_2^2 + A_{21} x_2 x_1 \tag{20}$$

The film and digital camera case study from Nilchiani et al.[44] was reformatted into the form of Equation 6. Using curve fitting of the market data from Ünver [14] (shown in Figure 3), the parameters were set so that $a = a_1 = a_2 = 0.37$, a positive value capturing the growth dynamics in the market, and $b = b_1 = b_2 = -1.6e^{-8}$, a negative value capturing the balancing dynamics in the market. The value for a was calculated by averaging the solutions for a_1 and a_2 and b was calculated by averaging the solutions for b_1 and b_2 found in Ünver [14]. This approach of setting a and b to be constant across all the technologies simplifies this method for finding the tipping point but gives each technology the same market-growth dynamics. This simplification works best when both technologies are competing in the same market. However, a good fit can potentially still be achieved, because the equations are under-constrained with multiple parameters that can be adjusted to match the actual market dynamics. As a result, this simplification of setting a constant a and b forces the adjacency matrix, A, to capture more of the predator-prey dynamics of the system. With x_1 modeling the dynamics of film cameras and x_2 modeling the dynamics of digital cameras, the elements of the adjacency matrix were varied using a Monte-Carlo simulation until the average percent error between the simulated results and actual data in Figure 3 was minimized. The simulation randomly set each element of A equal to a number between -5.0e^{-8} and 5.0e^{-8}, simulated the dynamics with the Runge-Kutta solver MATLAB ode45. The local optimum solution was selected from the minimum in the average percent error recorded across the runs. From this process, the following adjacency matrix provided the best fit for the actual market data.

$$A = \begin{bmatrix} 0 & -4.0e^{-8} \\ 2.8e^{-8} & 0 \end{bmatrix} \tag{21}$$

Figure 3 shows the simulated dynamics of this case in technology disruption versus the actual market data of film and digital camera sales for that period. The average percent error between the simulated results and actual data is 0.1952.

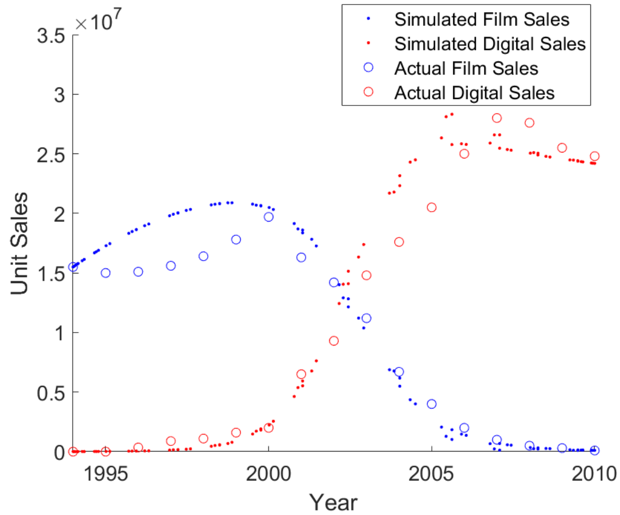

Figure 4. Simulated Disruption of the Camera Market with Simplified Self-Dynamics

One of the strengths of this modeling approach using network theory is that it scales to larger networks. To explore this ability of the model to scale, we expanded the model to capture the market dynamics of multiple technologies, not just two technologies. As discussed earlier, the camera industry was disrupted due to advances in the capabilities of cameras on cell phones. In essence, the use of cameras on cell phones has displaced much of the market that used to drive camera sales. This disruption is shown in figure 4, with cell phone ownership rising and digital camera ownership dropping from 2006 to 2019, following curves similar to the S-curves described by Christensen in figure 3. The model was expanded to include a third technology using Equation 6 and its variable definitions discussed above, repeated below as Equation 22.

$$\frac{dx_i}{dt} = a_i x_i + b_i x_i^2 + \sum_{j=1}^{n} A_{ij}\, x_i x_j \tag{22}$$

The film and digital market data were used from Ünver [14], and the cell phone data was obtained from public statistics websites [64, 65] and adjusted as follows. Since smartphones' turnover rate and lifespan were initially short and the technological improvements were rapid (also see Figure 2), the acquired data [64, 65] were converted into ownership instead of sales figures to provide a comparable basis. To conduct this conversion, the average lifespan of each product was accounted for at the time of its sale, and therefore, the total ownership numbers for each year could be calculated. This resulted in the progressions over time, as can be seen in Figure 5. With this approach, the smartphone sales numbers were considered in the total ownership numbers, with lifetime factors ranging from one year in 2007 to three years in 2020. This consideration means that a smartphone sold in 2007 was assumed to be used for one year, whereas a smartphone sold in 2017 would be used for an average of two years. All data is openly accessible through the cited sources, and the exact calculations can be provided upon request. The Monte-Carlo simulation discussed above was rerun, in this case randomly varying the a parameter between 0 and 0.5, the b parameter between 0 and -5e^{-10}, and the A matrix elements between -5.0e^{-8} and 5.0e^{-8}. Like before, the minimum average percent error was calculated between the simulated results and actual data shown in figure 5. The parameters that best fit this data are shown below. Note that in this case, the adjacency matrix was allowed to have both positive and negative values. Figure 5 shows that there is some error in the transient behavior of the simulated data compared to the actual data. The average percent error between the simulated results and actual data here is 1.6563.

$$a = a_1 = a_2 = a_3 = 0.2 \tag{23}$$

$$b = b_1 = b_2 = b_3 = -2.6e^{-10} \tag{24}$$

$$A = \begin{bmatrix} 0 & -4.2e^{-8} & 0 \\ 2.8e^{-8} & 0 & -1.2e^{-9} \\ 0 & -4.2e^{-8} & 0 \end{bmatrix} \tag{25}$$

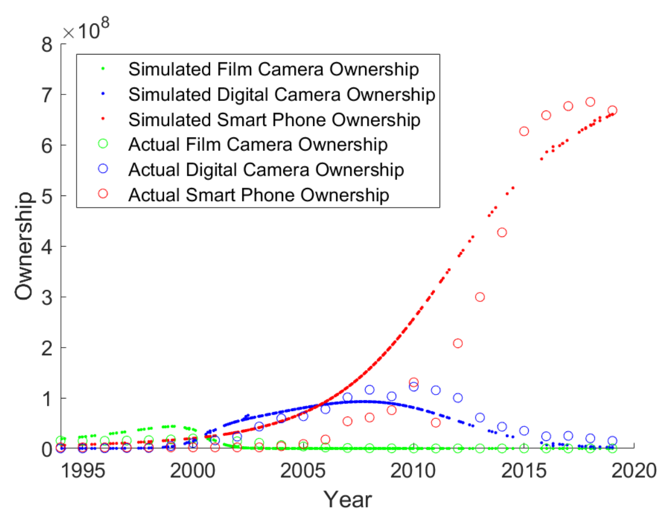

Figure 5. Simulated and actual camera and cell phone ownership

The result of the simulation in Figure 4 shows some deviations from the actual data related to film, digital cameras, and cell phone cameras, which may be due to several uncertainties, such as the financial crisis of 2008.

The separation of the predator and prey behaviors is done by separating the equation of the predator (cellphone) behavior from the equations of the more prey-like (camera) behavior to create $x_{eff,cell}$ and $x_{eff,cameras}$. The conversion into the effective plane is done with the following equations:

$$\beta_{eff} = \frac{1^T A s^{in}}{1^T A 1} \quad (26)$$

$$x_{eff,cameras} = \frac{1^T A \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}}{1^T A 1} \quad (27)$$

$$x_{eff,cell} = \frac{1^T A \begin{bmatrix} 0 \\ 0 \\ x_3 \end{bmatrix}}{1^T A 1} \quad (28)$$

Based on the described parameters and equations in this section, the simulations of the scenarios were analyzed. The following section will discuss the simulation results for the case study, outcomes, and insights.

5. RESULTS OF THE CASE STUDY AND DISCUSSIONS

Based on the parameters chosen in the case study to simulate the market dynamics and related data, the simulations were run to discover the tipping point at which disruptive technology takes over the incumbent one. The results are presented in this section.

Based on previous work on the determination of tipping points in ecological and supply-chain systems [12, 66-68], the authors propose a modified resilience index δ_{eff}^{c} defined as:

$$\delta_{eff}^{c} = \beta_{eff} - \beta_{eff}^{c} \quad (29)$$

The modified resilience index measures how far a disruptive or incumbent technology is from that tipping point, predicting whether a technology will succeed or fail.

In the digital versus film camera case study, 10,000 adjacency matrices were randomly generated while keeping a_1, a_2, b_1, and b_2 constant. Figure 5 exhibits the results, with the four possible β_{eff}^{c} tipping point plotted as red lines. Figure 5 also defines a region of the trade space that is confined between $\beta_{eff}^{c3} = 2b$ and $\beta_{eff}^{c0} = 0$ as two tipping points. The region between these two tipping points is where the incumbent technology can fail to maintain its market dominance and a region where the disruptive technology can fail to gain a foothold in the market. Figure 5 also consists of two simulation results; the top Figure shows the resiliency index versus sales number for the incumbent technology, and the bottom figure shows the resiliency index versus sales parameters of the disruptive technology. As can be seen in the region of trade space confined between $\beta_{eff}^{c3} = 2b$ and $\beta_{eff}^{c0} = 0$, the incumbent technology sales in various simulation results are higher than the disruptive technology, and therefore it defines a region in which the disruptive technology can fail. In two regions with resiliency index $\beta_{eff}^{c3} < 2b$ and $\beta_{eff}^{c0} > 0$, the disruptive technology has a much higher chance to succeed and dominate the market.

The historical case study of digital versus film cameras is marked in the simulation in Figure 5, which is clearly in the region of the trade space where disruptive technology can succeed. The case study of film vs. digital camera sales has a $\beta_{eff} = 1.87e^{-7}$, calculated using Equation 10 with the adjacency matrix in Equation 21. That system was clearly above the tipping point of $\beta_{eff}^{c0} = 0$ shown in Figure 6, so it was in the region where the disruptive technology succeeded. The calculation of the modified resilience index is then $\delta_{eff}^{c} = \beta_{eff} - \beta_{eff}^{c0} = 1.87e^{-7} - 0 = 1.87e^{-7}$. This result means that the system has δ_{eff}^{c} of distance between its current β_{eff} and β_{eff}^{c0}, the boundary where the system would enter the region where the disruptive technology could fail. Instead, the location of β_{eff} as shown in Figure 5 with an apparently large δ_{eff}^{c} between it and β_{eff}^{c0} means that this historic case study was solidly in the region where the disruptive technology succeeded. By comparison, the real-life result of that historic case was that the disruptive technology of digital cameras eventually dominated the market.

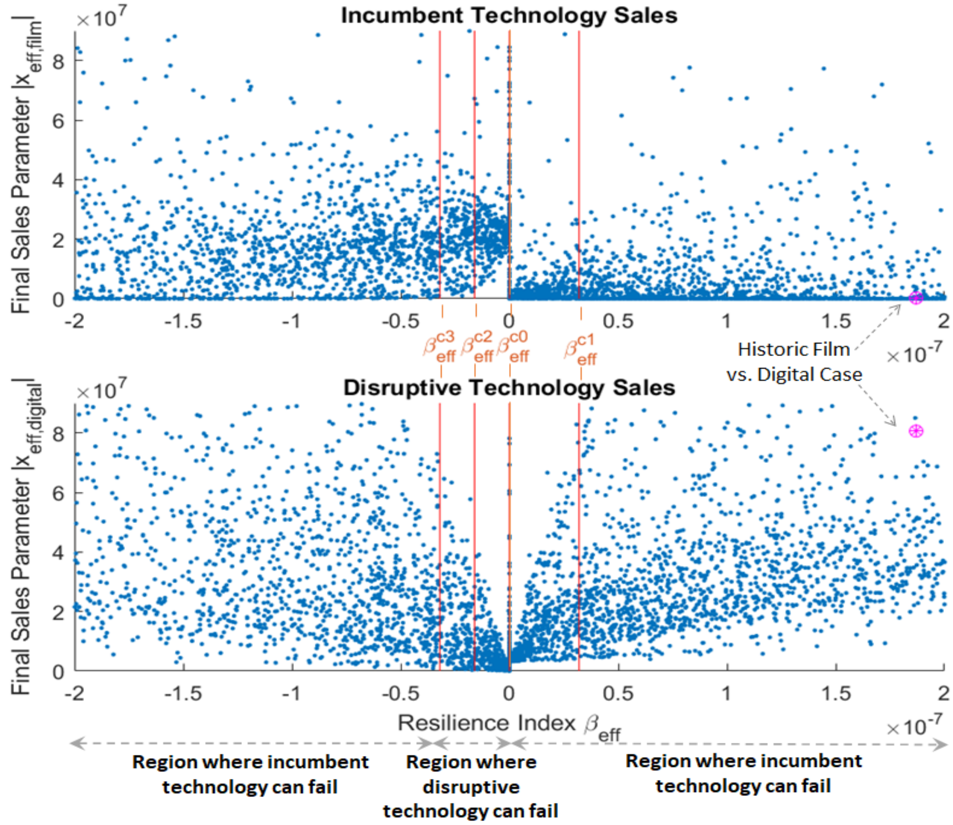

Figure 6. Tipping Points in the Success or Failure of a Disruptive Technology

The second part of the case study incorporates cell phones, following the same methodology as discussed in Section III. The predicted tipping points of this model are located at $\beta_{eff}^{c0} = 0$, $\beta_{eff}^{c1} = -2b$, $\beta_{eff}^{c2} = b$, and $\beta_{eff}^{c3} = 2b$. To populate various scenarios and the trade space we randomly generated 10,000 adjacency matrices, enabling both positive and negative values in A, and Figure 6 shows the results. For the case study of the cell phones vs. both types of cameras, the resilience index was calculated as $\beta_{eff} = 1.6e^{-7}$ and the final values for the camera and cell ownership parameters were $x_{eff,cameras} = 6.4e^{6}$ and $x_{eff,cell} = 5.6e^{7}$. These points are plotted in Figure 6 for reference.

The nature of the tipping points in this model is more complex than that of the model with only two technologies from Figure 5. The regions to the left and right of the tipping points are where the cell phones take over the market from the cameras, and the results of the case study that simulated the actual market dynamics from Figure 4 fit into that region, labeled with the case study marker. The scale of the resilience indices of these randomized case studies is much larger than the scale of the tipping points, such that the region between tipping points is not easily visible at this scale. However, if the dynamics had fallen into the region between the tipping points, then there are scenarios where the disruptive technology (cell phones) could fail, and the incumbent technology (cameras) would maintain market leadership, as shown in the zoomed views of figure 7.

The boundary that the disruptive-technology stakeholders would not want to cross is $\beta_{eff}^{c1} = -2b = 2.6e^{-10}$. The modified resilience index they would want to track would be $\delta_{eff}^{c} = \beta_{eff} - \beta_{eff}^{c1} = 1.6e^{-7} - 2.6e^{-10} = 1.5974e^{-7}$. If the market dynamics shifted so that δ_{eff}^{c} decreased significantly, then the incumbent technology is more likely to maintain a significant market presence. If δ_{eff}^{c} increases, then the incumbent technology is less likely to have continued success. In figure 7, the β_{eff} of the historic market is labeled at $\beta_{eff} = 1.6e^{-7}$, and its distance from β_{eff}^{c1} as measured by δ_{eff}^{c} is significant compared to the other simulated markets, showing that it is solidly in the region where the disruptive technology succeeds. Of note is that the labeled location of the ownership parameter for camera's, $x_{eff,cameras}$, is not zero. The survival of camera ownership is apparent in the real-life data of this historic case study because camera sales still continue at a lower level than their historic maximum as seen in Figure 5.

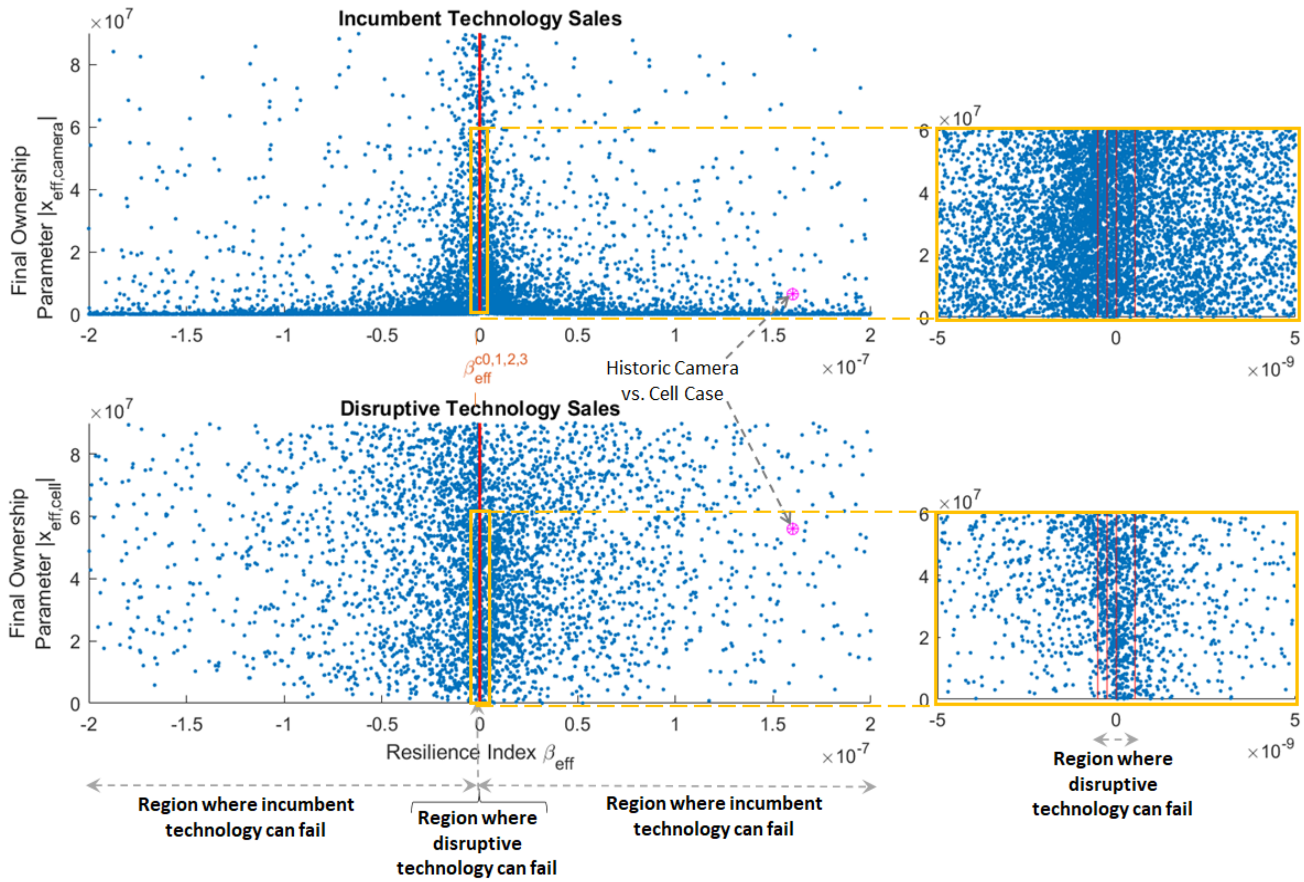

Figure 7. Tipping Points in Camera versus Cell Phone Market Dynamics

Looking at the simulation results, the authors argue that the model is capable of analyzing the inherent dynamics. The promise of this research lies in the analysis of potential scenarios to predict the potential for disruptiveness of a technology in the mainstream market. The suggested methodology and approach can help in the analysis of future novel technology development and dissemination. For example, the introduction of a new technology into a given market can be simulated with certain scenarios to evaluate the potential for the technology to disrupt the existing market. The market analysts and technology managers can begin by predicting scenarios that include various possible S-Curves patterns the market could take, and then use this method to calculate the modified resilience index, δ_{eff}^{c}, as a measurement of how disruptive each scenario could be. Such analyses are also not limited to a singular application but could be repeated iteratively over time, to track changes in δ_{eff}^{c} for potential and ongoing disruptions.

The developed model and the measures of the tipping point shed some light on the complex dynamics of an emerging (and potentially disruptive) technology within an established mainstream market along with evaluating its potential disruptiveness. This research also enables the analysis of influencing factors. This knowledge can then be used to improve the position of businesses in the market or as a factor to consider in the introduction of new technologies. This methodology can also be used for analyzing other systems that have predator-prey dynamics.

7. CONCLUSION

In this paper, a new model and methodology to identify the tipping point of a disruptive technology in market dynamics is developed, which defines whether a disruptive technology will succeed or fail. The contribution of the current study can be applied to both systems engineering as well as disruptive innovation management. Future work includes modifying this approach to a generalized framework for dynamically predicting the future success of potentially disruptive technology and testing its use in technology management as a tool for planning how to gain a market foothold for a technology, especially a disruptive technology. Future work can also include investigation into whether this method is more accurate when the dynamics equations are formulated so that the adjacency matrix elements are positive and constrained to decimal values between 0 and 1, as theorized in [12].

The current research results shed light on some of the dynamics of disruptive technologies and market behavior. In this paper, the methodology and case studies simulated are related to the saturated and developed markets [17]. The technologies in saturated markets have to compete for a finite number of resources in the form of demand/sales. If market saturation is not yet reached, resources to exploit would still be available, and less competition would exist. Competitors can expand into an unsaturated market, and several technologies can take a foothold with lower competition. In such cases, the methodology and formulations should be revised to capture unsaturated or new developing markets. Future work also includes the extension of the model to markets where two or more new technologies are introduced into the market in a short period of time.

As for the application of the methodology presented in this paper, various cases exist where this research, in its current form, can be applied to assess the disruptiveness of a technology. The application of this approach to changing market dynamics over time would help validate its use in market analysis processes. As part of a framework, this index can provide companies with valuable information to assess their position in the market regarding resiliency and the success of the new technology. The metric can be incorporated into risk frameworks for market shifts or investment analyses that assess novel technologies. The research presented by the authors has practical application and could enable risk reduction in the introduction of new technology into the mainstream market.

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APPENDIX

A	B	C	D	E	F	G
Year	Digital Camera Sales ($)	Average Life Span (Years)	Active Digital Cameras	Smartphones Sales ($)	Average Life Span (Years)	Active Smartphones
1999	5,057,576	16	5,057,576			
2000	10,819,583	16	15,877,159			
2001	15,955,813	16	31,832,972			
2002	23,365,320	16	55,198,292			
2003	43,392,510	16	98,590,802			
2004	59,404,649	16	157,995,451			
2005	63,575,997	16	221,571,448			
2006	77,632,502	16	299,203,950			
2007	100,981,778	16	400, 185,728	122,320,000	1	122,320,000
2008	116,166,909	16	516,352,637	139,290,000	1	139,290,000
2009	103,040,969	16	619,393,606	172,380,000	1	172,380,000
2010	121,766,943	16	741,160,549	296,650,000	1	296,650,000
2011	114,624,757	16	855,785,306	115,500,000	1	115,500,000
2012	100,374,356	16	956, 159,662	472,000,000	1.5	472,000,000
2013	61,005,309	16	1,017,164,971	680,110,000	1.5	916,110,000
2014	42,768,140	16	1,059,933,111	969,720,000	1.5	1,309,775,000
2015	35,215,670	16	1,090,091,205	1,423,900,000	1.7	1,908,760,000
2016	23,853,572	16	1,098,067,618	1,495,960,000	1.8	2,492,690,000
2017	25,088,712	16	1,091,323,358	1,536,540,000	2.2	2,733,308,000
2018	19,504,810	16	1,055,629,876	1,556,270,000	2.5	3,092,810,000
2019	14,862,729	16	971,901,803	1,517,830,000	3	3,381,408,000

Explanation:

The table above shows the active digital cameras and smartphones (with a camera) for the years 1999 through 2019. The active devices are calculated by using the annual sales and the average lifespan of the devices, as indicated in column C and F. After devices reach their expected life span maximum, they are considered out of use and are thus removed from the number of active devices. The average lifetime of a digital camera was assumed to be 16 years (since no replacement cycle data was available, so the development cycle time was used, which is close to 15 years [a]), while the lifespan of a smartphone ranges from 1 year in 2007 [b] to a peak of three years in 2019 [c].

a. https://the.me/camera-industrys-15-year-cycle/
b. https://www.businessinsider.com/the-smartphone-upgrade-cycle-2013-9
c. https://www.statista.com/statistics/619788/average-smartphone-life/